5-3 Study Guide and Intervention

Indirect Proof with Algebra

**Indirect Proof**
One way to prove that a statement is true is to assume that its conclusion is false and then show that this assumption leads to a contradiction of the hypothesis, a definition, postulate, theorem, or other statement that is accepted as true. That contradiction means that the conclusion cannot be false, so the conclusion must be true. This is known as **indirect proof**.

**Steps for Writing an Indirect Proof**
1. Assume that the conclusion is false.
2. Show that this assumption leads to a contradiction.
3. Point out that the assumption must be false, and therefore, the conclusion must be true.

**Example**
Given: $3x + 5 > 8$
Prove: $x > 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$3x + 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-3</td>
<td>-4</td>
</tr>
</tbody>
</table>

Step 1 Assume that $x$ is not greater than 1. That is, $x = 1$ or $x < 1$.
Step 2 Make a table for several possibilities for $x = 1$ or $x < 1$. The contradiction is that when $x = 1$ or $x < 1$, then $3x + 5$ is not greater than 8.
Step 3 This contradicts the given information that $3x + 5 > 8$. The assumption that $x$ is not greater than 1 must be false, which means that the statement "$x > 1$" must be true.

**Exercises**
Write the assumption you would make to start an indirect proof of each statement.

1. If $2x > 14$, then $x > 7$. $x \leq 7$
2. For all real numbers, if $a + b > c$, then $a > c - b$. $a \leq c - b$

Complete the proof.
Given: $n$ is an integer and $n^2$ is even.
Prove: $n$ is even.

3. Assume that $n$ is not even. That is, assume $n$ is odd.
4. Then $n$ can be expressed as $2a + 1$ by the meaning of odd number.
5. $n^2 = (2a + 1)^2$
   - Substitution
6. $= (2a + 1)(2a + 1)$
   - Multiply.
7. $= 4a^2 + 4a + 1$
   - Simplify.
8. $= 2(2a^2 + 2a) + 1$
   - **Distributive Property**
9. $2(2a^2 + 2a) + 1$ is an odd number. This contradicts the given that $n^2$ is even, so the assumption must be false.
10. Therefore, $n$ is even.

5-3 Study Guide and Intervention (continued)

**Indirect Proof with Geometry**
To write an indirect proof in geometry, you assume that the conclusion is false. Then you show that the assumption leads to a contradiction. The contradiction shows that the conclusion cannot be false, so it must be true.

**Example**
Given: $m\angle C = 100$
Prove: $\angle A$ is not a right angle.

**Step 1**
Assume that $\angle A$ is a right angle.

**Step 2**
Show that this leads to a contradiction. If $\angle A$ is a right angle, then $m\angle A = 90$ and $m\angle C + m\angle A = 100 + 90 = 190$. Thus the sum of the measures of the angles of $\triangle ABC$ is greater than 180.

**Step 3**
The conclusion that the sum of the measures of the angles of $\triangle ABC$ is greater than 180 is a contradiction of a known property. The assumption that $\angle A$ is a right angle must be false, which means that the statement "$\angle A$ is not a right angle" must be true.

**Exercises**
Write the assumption you would make to start an indirect proof of each statement.

1. If $m\angle A = 90$, then $m\angle B = 45$. $m\angle B \neq 45$
2. If $\overline{AB}$ is not congruent to $\overline{EF}$, then $\triangle ABE$ is not isosceles. $\triangle ABE$ is isosceles.

Complete the proof.
Given: $\angle 1 \cong \angle 2$ and $DG$ is not congruent to $FG$.
Prove: $DE$ is not congruent to $FE$.

3. Assume that $DE \cong FE$. Assume the conclusion is false.
4. $EG \cong EG$ **Reflexive Property**
5. $\triangle EDG \cong \triangle EFG$ **SAS**
6. $DG \cong FG$ **CPCTC**
7. This contradicts the given information, so the assumption must be false.
8. Therefore, $DE$ is not congruent to $FE$. 

 answered (lesson 5-3)
5-3 Skills Practice
Indirect Proof

Write the assumption you would make to start an indirect proof of each statement.

1. \( m \angle ABC < m \angle CBA \)
   \( m \angle ABC = m \angle CBA \)

2. \( \triangle DEF = \triangle RST \)
   \( \triangle DEF \neq \triangle RST \)

3. Line \( a \) is perpendicular to line \( b \).
   Line \( a \) is not perpendicular to line \( b \).

4. \( \angle 5 \) is supplementary to \( \angle 6 \).
   \( \angle 5 \) is not supplementary to \( \angle 6 \).

PROOF Write an indirect proof.

5. Given: \( x^2 + 8 \leq 12 \)
   Prove: \( x \leq 2 \)

Proof:
Step 1: Assume \( x > 2 \).
Step 2: If \( x > 2 \), then \( x^2 > 4 \). But if \( x^2 > 4 \), it follows that \( x^2 + 8 > 12 \).
   This contradicts the given fact that \( x^2 + 8 \leq 12 \).
Step 3: Since the assumption of \( x > 2 \) leads to a contradiction, it must be false. Therefore, \( x \leq 2 \) must be true.

6. Given: \( \angle D \neq \angle F \).
   Prove: \( DE \neq EF \)

Proof:
Step 1: Assume \( DE = EF \).
Step 2: If \( DE = EF \), then \( \triangle DEF \) by the definition of congruent segments.
   But if \( DE = EF \), then \( \angle D = \angle F \) by the Isosceles Triangle Theorem.
   This contradicts the given information that \( \angle D \neq \angle F \).
Step 3: Since the assumption that \( DE = EF \) leads to a contradiction, it must be false. Therefore, it must be true that \( DE \neq EF \).

5-3 Practice (Average)
Indirect Proof

Write the assumption you would make to start an indirect proof of each statement.

1. \( BD \) bisects \( \angle ABC \).
   \( BD \) does not bisect \( \angle ABC \).

2. \( RT = TS \)
   \( RT \neq TS \)

PROOF Write an indirect proof.

3. Given: \( -4x + 2 < -10 \)
   Prove: \( x > 3 \)

Proof:
Step 1: Assume \( x \leq 3 \).
Step 2: If \( x \leq 3 \), then \(-4x + 2 \geq -10 \). This contradicts the given inequality.
Step 3: Since the assumption that \( x \leq 3 \) leads to a contradiction, it must be true that \( x > 3 \).

4. Given: \( m \angle 2 + m \angle 3 = 180 \)
   Prove: \( a \parallel b \)

Proof:
Step 1: Assume \( a \parallel b \).
Step 2: If \( a \parallel b \), then the consecutive interior angles \( \angle 2 \) and \( \angle 3 \) are supplementary. Thus \( m \angle 2 + m \angle 3 = 180 \). This contradicts the given statement that \( m \angle 2 + m \angle 3 = 180 \).
Step 3: Since the assumption leads to a contradiction, the statement \( a \parallel b \) must be false. Therefore, \( a \parallel b \) must be true.

5. PHYSICS Sound travels through air at about 344 meters per second when the temperature is 20°C. If Enrique lives 2 kilometers from the fire station and it takes 5 seconds for the sound of the fire station siren to reach him, how can you prove indirectly that it is not 20°C when Enrique hears the siren?

Assume that it is 20°C when Enrique hears the siren, then show that at this temperature it will take more than 5 seconds for the sound of the siren to reach him. Since the assumption is false, you will have proved that it is not 20°C when Enrique hears the siren.